

Essay on the Non-Maxwellian Theories of Electromagnetism*

Valeri V. Dvoeglazov

*Escuela de Física, Universidad Autónoma de Zacatecas
Antonio Dovalí Jaime s/n, Zacatecas 98068, ZAC., México
Internet address: VALERI@CANTERA.REDUAZ.MX
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In the first part of this paper we review several formalisms which give alternative ways for describing the light. They are: the formalism ‘baroque’ and the Majorana-Oppenheimer form of electrodynamics, the Sachs’ theory of Elementary Matter, the Dirac-Fock-Podol’sky model, its development by Staruszkiewicz, the Evans-Vigier $\mathbf{B}^{(3)}$ field, the theory with an invariant evolution parameter of Horwitz, the analysis of the action-at-a-distance concept, presented recently by Chubykalo and Smirnov-Rueda, and the analysis of the claimed ‘longitudinality’ of the antisymmetric tensor field after quantization. The second part is devoted to the discussion of the Weinberg formalism and its recent development by Ahluwalia and myself.

I. HISTORICAL NOTES

The Maxwell’s electromagnetic theory perfectly describes many observed phenomena. The accuracy in predictions of the quantum electrodynamics is without precedents [22]. They are widely accepted as the only tools to deal with electromagnetic phenomena. Other modern field theories have been built on the basis of the similar principles to deal with weak, strong and gravitational interactions. Nevertheless, many scientists felt some unsatisfactions with both these theories since almost their appearance, see, *e.g.*, refs. [74] and refs. [19,46,59]. In the preface to the Dover edition of his book [10] A. Barut writes (1979): “Electrodynamics and the classical theory of fields remain very much alive and continue to be the source of inspiration for much of the modern research work in new physical theories” and in the preface to the first edition he said about shortcomings in the modern quantum field theory. They are well known. Furthermore, in spite of much expectation in the sixties and the seventies after the proposal of the Glashow-Salam-Weinberg model and the quantum chromodynamics, elaboration of the unified field theory, based on the gauge principle, has come across with serious difficulties. In the end of the nineties there are a lot of experiments in our disposition, which do not find satisfactory explanations on the basis of the standard model. First of all, one can single out the following ones: the LANL neutrino oscillation experiment; the atmospheric neutrino anomaly, the solar neutrino puzzle (all of the above-mentioned imply existence of the neutrino mass); the tensor coupling in decays of π^- and K^+ mesons; the dark matter problem; the observed periodicity of the number distribution of galaxies, and the ‘spin crisis’ in QCD. Furthermore, experiments and observations concerning with superluminal phenomena: negative mass-square neutrinos, tunnelling photons, “X-shaped waves” and superluminal expansions in quasars and in *galactic* objects.

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In the meantime almost since the proposal of the Lorentz-Poincaré-Einstein theory of relativity [31] and the development of the mathematical formalism of the Poincaré group [73] several physicists (including A. Einstein, W. Pauli and M. Sachs) thought that in order to build a reliable theory, based on relativistic ideas, one must utilize the irreducible representations of the underlying symmetry group — the Poincaré group of special relativity, *i.e.* to build it on the first principles. Considerable efforts in this direction have been recently undertaken by M. Evans. Since the prediction and the discovery of an additional phase-free variable, the spin, which all observed fundamental particles have, to find its classical analogue and to relate it with known fields and/or space-time structures (perhaps, in higher dimensions) was one of the important tasks of physicists. We can say now that this has been done (see papers and books of M. Evans and what is below). In the end of this introductory part we note that while the ‘Ultimate’ Theory has not yet been proposed recent papers of D. V. Ahluwalia, M. W. Evans, E. Recami and several other works provide a sufficiently clear way to this goal.

We deal below with the historical development, with the ideas which could be useful to proceed further.

E = 0 solutions. First of all, I would like to mention the problem with existence of ‘acausal’ solutions of relativistic wave equations of the first order. In ref. [1] and then in [2] the author found that massless equations of the form¹

$$(\mathbf{J} \cdot \mathbf{p} + p_0 \mathbb{1}) \phi_R(\mathbf{p}) = 0 \quad , \quad (1a)$$

$$(\mathbf{J} \cdot \mathbf{p} - p_0 \mathbb{1}) \phi_L(\mathbf{p}) = 0 \quad (1b)$$

have acausal dispersion relations, see Table 2 in [1]. In the case of the spin $j = 1$ this manifests in existence of the solution with the energy $E = 0$. Some time ago we learned that the same problem has been discussed by J. R. Oppenheimer [58], S. Weinberg [71b] and E. Gianetto [49c]. For instance, Weinberg on p. 888 indicated that “for $j = 1/2$ [the equations (1a,1b)] are the Weyl equations for the left- and right-handed neutrino fields, while for $j = 1$ they are just Maxwell’s free-space equations for left- and right-circularly polarized radiation:

$$\nabla \times [\mathbf{E} - i\mathbf{B}] + i(\partial/\partial t)[\mathbf{E} - i\mathbf{B}] = 0, \quad (2a)$$

$$\nabla \times [\mathbf{E} + i\mathbf{B}] - i(\partial/\partial t)[\mathbf{E} + i\mathbf{B}] = 0. \quad (2b)$$

The fact that these field equations are of first order for any spin seems to me to be of no great significance, since in the case of massive particles we can get along perfectly well with $(2j+1)$ – component fields which satisfy only the Klein-Gordon equation.” This is obviously a remarkable and bold conclusion of the great physicist. In the rest of the paper we try to understand it.

Oppenheimer concerns with the $E = 0$ solution on the pages 729, 730, 733 (see also the discussion on p. 735) and indicated at its connection with the electrostatic solutions of Maxwell’s equations. “In the absence of charges there may be no such field.” This is contradictory: free-space Maxwell’s equations do not contain terms ρ_e or ρ_m , the charge densities, but dispersion relations still tell us about the solution $E = 0$. He deals further with the matters of relativistic invariance of the matrix equation (p. 733) and suggests that

¹Here and below in this historical section we try to keep the notation and the metric of original papers.

the components of ψ ($\phi_{R,L}$ in the notation of [1,2]) transform under a pure Lorentz transformations like the space components of a covariant 4-vector. This induces him to extend the matrices and the wave functions to include the forth component. Similar formulation has been developed by Majorana [49]. If so, it would be already difficult to consider $\phi_{R,L}$ as Helmholtz' bivectors because they have different laws for pure Lorentz transformations. What does the 4-component function (and its space components) corresponds to? Finally, he indicated (p. 728) that $c\boldsymbol{\tau}$, the angular momentum matrices, and the corresponding density-flux vector may “play in some respects the part of the velocity”, with eigenvalues $0, \pm c$. So, the formula (5) of the paper [58] may have some relations with the discussion of the convection displacement current in [17], see below.

Finally, M. Moshinsky and A. Del Sol found the solution of the similar nature in a two-body relativistic problem [53]. Of course, it is connected with earlier considerations, *e.g.*, in the quasipotential approach. In order to try to understand the physical sense of the $E = 0$ solutions and the corresponding field components let us consider other generalizations of the Maxwell's formalism.

The formalism ‘baroque’. In this formalism proposed in the fifties by K. Imaeda [41] and T. Ohmura [57], who intended to solve the problem of stability of an electron, additional scalar and pseudoscalar fields are introduced in the Maxwell's theory. Monopoles and magnetic currents are also present in this theory. The equations become:

$$\text{rot } \mathbf{H} - \partial \mathbf{E} / \partial x_0 = \mathbf{i} - \text{grad } e \quad , \quad (3a)$$

$$\text{rot } \mathbf{E} + \partial \mathbf{H} / \partial x_0 = \mathbf{j} + \text{grad } h \quad , \quad (3b)$$

$$\text{div } \mathbf{E} = \rho + \partial e / \partial x_0 \quad , \quad (3c)$$

$$\text{div } \mathbf{H} = -\sigma + \partial h / \partial x_0 \quad . \quad (3d)$$

“Each of \mathbf{E} and \mathbf{H} is separated into two parts $\mathbf{E}^{(1)} + \mathbf{E}^{(2)}$ and $\mathbf{H}^{(1)} + \mathbf{H}^{(2)}$: one is the solution of the equations with \mathbf{j}, σ, h zero, and other is the solution of the equations with \mathbf{i}, ρ, e zero.” Furthermore, Dra. T. Ohmura indicated at existence of longitudinal photons in her model: “It will be interesting to test experimentally whether the γ -ray keeps on its transverse property even in the high energy region as derived from the Maxwell theory or it does not as predicted from our hypothesis.” In fact, the equations (3a-3d) can be written in a matrix notation, what leads to the known Majorana-Oppenheimer formalism for the $(0,0) \oplus (1,0)$ (or $(0,0) \oplus (0,1)$) representation of the Poincaré group [49,58], see also [21]. In a form with the Majorana-Oppenheimer matrices

$$\rho^1 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \quad , \quad \rho^2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & i \\ -1 & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{pmatrix} \quad , \quad (4a)$$

$$\rho^3 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad , \quad \rho^0 = \mathbb{1}_{4 \times 4} \quad , \quad (4b)$$

and $\bar{\rho}^0 \equiv \rho^0$, $\bar{\rho}^i \equiv -\rho^i$ the equations without an explicit mass term are written

$$(\rho^\mu \partial_\mu) \psi_1(x) = \phi_1(x) \quad , \quad (5a)$$

$$(\bar{\rho}^\mu \partial_\mu) \psi_2(x) = \phi_2(x) \quad . \quad (5b)$$

The ϕ_i are the “quadrivectors” of the sources

$$\phi_1 = \begin{pmatrix} -\rho + i\sigma \\ i\mathbf{j} - \mathbf{i} \end{pmatrix} \quad , \quad \phi_2 = \begin{pmatrix} \rho + i\sigma \\ -i\mathbf{j} - \mathbf{i} \end{pmatrix} \quad . \quad (6)$$

The field functions are

$$\psi_1(p^\mu) = \mathcal{C}\psi_2^*(p^\mu) = \begin{pmatrix} -i(E^0 + iB^0) \\ E^1 + iB^1 \\ E^2 + iB^2 \\ E^3 + iB^3 \end{pmatrix} \quad , \quad \psi_2(p^\mu) = \mathcal{C}\psi_1^*(p^\mu) = \begin{pmatrix} -i(E^0 - iB^0) \\ E^1 - iB^1 \\ E^2 - iB^2 \\ E^3 - iB^3 \end{pmatrix} \quad , \quad (7)$$

where $E^0 \equiv -h$, $B^0 \equiv e$ and

$$\mathcal{C} = \mathcal{C}^{-1} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad , \quad \mathcal{C}\alpha^\mu\mathcal{C}^{-1} = \bar{\alpha}^{\mu *} \quad . \quad (8)$$

When sources are switched off the equations have relativistic dispersion relations $E = \pm|\mathbf{p}|$ only. In ref. [49] zero-components of ψ have been connected with $\pi_0 = \partial_\mu A^\mu$, the zero-component of the canonically conjugate momentum to the field A_μ . H. E. Moses developed the Oppenheimer's idea [58] that the longitudinal part of the electromagnetic field is connected somehow with the sources which created it [52, Eq.(5.21)]. Moreover, it was mentioned in this work that even after the switchoff of the sources, the localized field can possess the longitudinal component (*Example 2*). Then, he made a convention which, in my opinion, is required to give more rigorous scientific basis: "... ψ^A is not suitable for a final field because it is not purely transverse. Hence we shall subtract the part whose divergence is not zero."

Finally, one should mention ref. [48], the proposed formalism is connected with the formalism of the previously cited works (and with the massive Proca theory) . Two of Maxwell's equations remain unchanged, but one has additional terms in two other ones:

$$\nabla \times \mathbf{H} - \partial \mathbf{D} / \partial t = \mathbf{J} - (1/\mu_0 l^2) \mathbf{A} \quad , \quad (9a)$$

$$\nabla \cdot \mathbf{D} = \rho - (\epsilon_0/l^2) V \quad , \quad (9b)$$

where l is of the dimensions length and is suggested by Lyttleton and Bondi to be of the order of the radius of the Universe. \mathbf{A} and V are the vector and scalar potentials, which put back into two Maxwell's equations for strengths. So, these additional terms contain information about possible effects of the photon mass. This was applied to explain the expansion of the Universe. The Watson's generalization, also discussed in [48b], is based on the introduction of the additional gradient current (as in Eqs. (3a,3c)) and, in fact, repeats in essence the Majorana-Oppenheimer and Imaeda-Ohmura formulations. On a scale much smaller than a radius of the Universe, both formulations were shown by Chambers to be equivalent. The difference obtained is of order l^{-2} at the most. In fact, both formulations were noted by Chambers to contain local creation of the charge.²

²The question of the integral conservation of the charge over the volume still deserves elaboration, the question of possibility to observe such a type of non-conservation as well. These questions may be connected with the boundary conditions on the sphere of the radius l .

The theory of Elementary Matter. The formalism proposed by M. Sachs [60,61] is on the basis of the consideration of spinorial functions composed of 3-vector components:

$$\phi_1 = \begin{pmatrix} G_3 \\ G_1 + iG_2 \end{pmatrix} \quad , \quad \phi_2 = \begin{pmatrix} G_1 - iG_2 \\ -G_3 \end{pmatrix} \quad , \quad (10)$$

where $G_k = H_k + iE_k$ ($k = 1, 2, 3$). 2-component functions of the currents are constructed in the following way:

$$\Upsilon_1 = -4\pi i \begin{pmatrix} \rho + j_3 \\ j_1 + ij_2 \end{pmatrix} \quad , \quad \Upsilon_2 = -4\pi i \begin{pmatrix} j_1 - ij_2 \\ \rho - j_3 \end{pmatrix} \quad . \quad (11)$$

The dynamical equation in this formalism reads

$$\sigma^\mu \partial_\mu \phi_\alpha = \Upsilon_\alpha \quad . \quad (12)$$

“...Eq. (12) is *not* equivalent to the less general form of Maxwell’s equations. That is to say the spinor equations (12) are not merely a rewriting of the vector form of the field equations, they are a true generalization in the sense of transcending the predictions of the older form while also agreeing with all of the correct predictions of the latter ... Eq. (12) may be rewritten in the form of four conservation equations $\partial_\mu(\phi_\alpha^\dagger \sigma^\mu \phi_\beta) = \phi_\alpha^\dagger \Upsilon_\beta + \Upsilon_\alpha^\dagger \phi_\beta$ [which] entails eight real conservation laws.” For instance, these equations could serve as a basis for describing parity-violating interactions [60a], and can account for the spin-spin interaction as well from the beginning [60d,p.934]. The formalism was applied to explain several puzzles in neutrino physics. The connection with the Pauli Exclusion Principle was revealed. The theory, when the interaction (‘matter field labeling’) is included, is essentially bi-local.³ “What was discovered in this research program, applied to the particle-antiparticle pair, was that an exact solution for the coupled field equations for the pair, in its rest frame, gives rise (from Noether’s theorem) to a prediction of null energy, momentum and angular momentum, when it is in this particular *bound state* [61].” Later [61] this type of equations was written in the quaternion form with the continuous function $m = \lambda\hbar/c$ identified with the inertial mass. Thus, an extension of the model to the general relativity case was proposed. Physical consequences of the theory are: a) the formalism predicts while small but non-zero masses and the infinite spectrum of neutrinos; b) the Planck spectral distribution of black body radiation follows; c) the hydrogen spectrum (including the Lamb shift) can be deduced; d) grounds for the charge quantization are proposed; e) the lifetime of the muon state was predicted; f) the electron-muon mass splitting was discussed, “the difference in the mass eigenvalues of a doublet depends on the alteration of the geometry of space-time in the vicinity of excited pairs of the “physical vacuum” [“a degenerate gas of spin-zero objects”, longitudinal and scalar photons, in fact – my comment] — leading, in turn, to a dependence of the ratio of mass eigenvalues on the fine-structure constant”. That was impressive work and these are impressive results!

Quantum mechanics of the phase. A. Staruszkiewicz [63,64] considers the Lagrangian and the action of a potential formulation for the electromagnetic field, which include a longitudinal part:

³The hypothesis of the non-local nature of the charge seems to have been firstly proposed by J. Frenkel.

$$\mathcal{S} = -\frac{1}{16\pi} \int d^4x \left\{ F_{\mu\nu} F^{\mu\nu} + 2\gamma \left(\partial^\mu A_\mu + \frac{1}{e} \square S \right)^2 \right\} . \quad (13)$$

S is a scalar field called the phase. As a matter of fact, this formulation was shown to be a development of the Dirac-Fock-Podol'sky model in which the current is a gradient of some scalar field [34]:

$$4\pi j_\nu = -\partial_\nu F \quad . \quad (14)$$

The modified Maxwell's equations are written:

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0 \quad , \quad (15a)$$

$$\partial^\mu F_{\mu\nu} + \partial_\nu F = 0 \quad . \quad (15b)$$

Again we see a gradient current and, therefore, the Dirac-Fock-Podol'sky model is a simplified version (seems, without monopoles) of the more general Majorana-Oppenheimer theory. Staruszkiewicz put forth the questions [64], see also [57b] and [35]: “Is it possible to have a system, whose motion is determined completely by the charge conservation law alone? Is it possible to have a pure charge not attached to a nonelectromagnetic piece of matter?” and answering came to the conclusion “that the Maxwell electrodynamics of a gradient current is a closed dynamical system.” The interpretation of a scalar field as a phase of the expansion motion of a charge under repulsive electromagnetic forces was proposed. “They [the Dirac-Fock-Podol'sky equations] describe a charge let loose by removal of the Poincaré stresses.” The phase was then related with the vector potential by means of [64e,p.902]⁴

$$S(x) = -e \int A_\mu(x-y) j^\mu(y) d^4y \quad , \quad \partial_\mu j^\mu(y) = \delta^{(4)}(y) \quad . \quad (16)$$

The operator of a number of zero-frequency photons was studied. The total charge of the system, found on the basis of the Nöther theorem, was connected with the change of the phase between the positive and the negative time-like infinity: $Q = -\frac{e}{4\pi} [S(+\infty) - S(-\infty)]$. It was shown that e^{iS} , having a Bose-Einstein statistics, can serve itself as a creation operator: $Qe^{iS}|0\rangle = [Q, e^{iS}]|0\rangle = -e e^{iS}|0\rangle$. Questions of fixing the factor γ by appropriate physical conditions were also answered. Finally, the Coulomb field was decomposed into irreducible unitary representations of the proper orthochronous Lorentz group [65]. Both representations of the main serie and the supplementary serie were regarded. In my opinion, these researches can help to understand the nature of the charge and of the fine structure constant.

Invariant evolution parameter. The theory of electromagnetic field with an invariant evolution parameter (τ , the Newtonian time) has been worked out by L. P. Horwitz [38–40]. It is a development of the Stueckelberg formalism [66] and I consider this theory as an important step to understanding the nature of our space-time. The Stueckelberg equation:

$$i \frac{\partial \psi_\tau(x)}{\partial \tau} = K \psi_\tau(x) \quad (17)$$

⁴The formula (16) is reminiscent to the Barut self-field electrodynamics [9]. This should be investigated by taking 4-divergence of the Barut's *ansatz*.

is deduced on the basis of his worldline classical relativistic mechanics with following setting up the covariant commutation relations $[x^\mu, p^\nu] = ig^{\mu\nu}$. Remarkably that he proposed a classical analogue of antiparticle (which, in fact, has been later used by R. Feynman) and of annihilation processes. As noted by Horwitz if one insists on the $U(1)$ gauge invariance of the theory based on the Stueckelberg-Schrödinger equation (17) one arrives at the 5-potential electrodynamics ($i\partial_\tau \rightarrow i\partial_\tau + e_0 a_5$) where the equations, which are deduced by means of the variational principle, read

$$\partial_\beta f^{\alpha\beta} = j^\alpha \quad (18)$$

($\alpha, \beta = 1 \dots 5$), with an additional fifth component of the conserved current $\rho = |\psi_\tau(x)|^2$. The underlying symmetry of the theory can be $O(3, 2)$ or $O(4, 1)$ “depending on the choice of metric for the raising and lowering of the fifth (τ) index [38]”. For Minkowski-space components the equation (18) is reduced to $\partial_\nu f^{\mu\nu} + \partial_\tau f^{\mu 5} = j^\mu$. The Maxwell’s theory is recovered after integrating over τ from $-\infty$ to ∞ , with appropriate asymptotic conditions. The formalism has been applied mainly in the study of the many-body problem and in the measurement theory, namely, bound states (the hydrogen atom), the scattering problem, the calculation of the selection rules and amplitudes for radiative decays, a covariant Zeeman effect, the Landau-Peierls inequality. Two crucial experiments which may check validity and may distinguish the theory from ordinary approaches have also been proposed [40, p.15].

Furthermore, one should mention that in the framework of the special relativity version of the Feynman-Dyson proof of the Maxwell’s equations [30] S. Tanimura came to unexpected conclusions [68] which are related with the formulation defended by L. Horwitz. Trying to prove the Maxwell’s formalism S. Tanimura arrived at the conclusion about a theoretical possibility of its generalization. According to his consideration the 4-force acting on the particle in the electromagnetic field must be expressed in terms of

$$F^\mu(x, \dot{x}) = G^\mu(x) + \langle F^\mu{}_\nu(x) \dot{x}^\nu \rangle \quad , \quad (19)$$

where the symbol $\langle \dots \rangle$ refers to the Weyl-ordering prescription. The fields $G^\mu(x)$, $F^\mu{}_\nu(x)$ satisfy⁵

$$\partial_\mu G_\nu - \partial_\nu G_\mu = 0 \quad , \quad (20a)$$

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \quad . \quad (20b)$$

This implies that apart from the 4-vector potential $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ there exists a scalar field $\phi(x)$ such that $G_\mu = \partial_\mu \phi$. One may try to compare this result with the fact of existence of additional scalar field components in the Majorana-Oppenheimer formulation of electrodynamics and with the Stueckelberg-Horwitz theory. The latter has been done by Prof. Horwitz himself [38c] by the identification $F_{\mu 5} = -F_{5\mu} = G_\mu$ and the explicit demonstration that for the off-shell theory the Tanimura’s equations reduce to

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\sigma} + \partial_\rho F_{\mu\nu} = 0 \quad , \quad (21a)$$

$$\partial_\mu G_\nu - \partial_\nu G_\mu + \frac{\partial F_{\mu\nu}}{\partial \tau} = 0 \quad , \quad (21b)$$

$$m\ddot{x}^\mu = G^\mu(\tau, x) + F^{\mu\nu}(\tau, x)\dot{x}_\nu \quad . \quad (21c)$$

⁵Of course, one can repeat the Tanimura proof for dual fields and obtain two additional equations.

Finally, among theories with additional parameters one should mention the quantum field model built in the de Sitter momentum space $p_5^2 - p_4^2 - p_3^2 - p_2^2 - p_1^2 = M^2$, ref. [44]. The parameter M is considered as a new physical constant, the fundamental mass. In a configurational space defined on the basis of the Shapiro transformations the equations become the finite-difference equations thus leading to the lattice structure of the space. In the low-energy limit ($M \rightarrow \infty$) the theory is equivalent to the standard one.

Action-at-a-distance. In the paper [16] A. E. Chubykalo and R. Smirnov-Rueda revealed on the basis of the analysis of the Cauchy problem of the D'Alembert and the Poisson equations that one should revive the concept of the *instantaneous* action-at-a-distance in classical electrodynamics in order to remove some inconsistencies of the description by means of the Liénard-Wiechert potentials. The essential feature of the formalism is in introduction of two types of field functions, with the explicit and implicit dependences on time. The energy of longitudinal modes in this formulation cannot be stored locally in the space, the spread velocity may be whatever and so one has also $E = 0$. The new convection displacement current was proposed in [17] on the basis of the development of this wisdom. It has a form $j_{\text{disp}} = -\frac{1}{4\pi}(\mathbf{v} \cdot \nabla)\mathbf{E}$. This is a resurrection of the Hertz' ideas (later these ideas have been defended by T. E. Phipps, jr.) to replace the partial derivative by the total derivative in the Maxwell's equations. In my opinion, one can also reveal connections with the Majorana-Oppenheimer formulation following to the analysis of ref. [58, p.728].

F. Belinfante [11a] appears to come even earlier to the Sachs' idea about the "physical vacuum" as pairs of some particles from a very different viewpoint. In his formulation of the quantum-electrodynamic perturbation theory zero-order approximation is determined in which scalar and longitudinal photons are present in pairs. He also considered [11b] the Coulomb problem in the frameworks of the quantum electrodynamics and proved that the signal can be transmitted with the velocity greater than c . So, this old work appears to be in accordance with recent experimental data (particularly, with the claims of G. Nimtz *et al.* about a wave packet propagating faster than c through a barrier, which was used "to transmit Mozart's Symphony No. 40 through a tunnel of 114 mm length at a speed of $4.7c$ "). As indicated by E. Recami in a private communication the $E = 0$ solutions can be put in correspondence to a tachyon of the infinite velocity.

Evans-Vigier $\mathbf{B}^{(3)}$ field. In a recent serie of remarkable papers (in FPL, FP, Physica A and B, Nuovo Cimento B) and books M. Evans and J.-P. Vigiér indicated the possibility of consideration of the longitudinal $\mathbf{B}^{(3)}$ field for describing many electromagnetic phenomena and in cosmological models as well [32]. It is connected with transversal modes

$$\mathbf{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}}(i\mathbf{i} + \mathbf{j})e^{i\phi} \quad , \quad (22a)$$

$$\mathbf{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}}(-i\mathbf{i} + \mathbf{j})e^{-i\phi} \quad , \quad (22b)$$

$\phi = \omega t - \mathbf{k} \cdot \mathbf{r}$, by means of the cyclic relations

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*} \quad , \quad (23a)$$

$$\mathbf{B}^{(2)} \times \mathbf{B}^{(3)} = iB^{(0)}\mathbf{B}^{(1)*} \quad , \quad (23b)$$

$$\mathbf{B}^{(3)} \times \mathbf{B}^{(1)} = iB^{(0)}\mathbf{B}^{(2)*} \quad . \quad (23c)$$

The indices (1), (2), (3) denote components of the vector in the circular basis and, thus, the longitudinal field $\mathbf{B}^{(3)}$ presents itself a third component of the 3-vector in some isovector space. "The conventional $O(2)$ gauge geometry is replaced by a non-Abelian $O(3)$ gauge

geometry and the Maxwell equations are thereby generalized” in this approach. Furthermore, some success in the problem of the unification of gravitation and electromagnetism has been achieved in recent papers by M. Evans. It was pointed out by several authors, *e.g.*, [33,28] that this field is the simplest and most natural (classical) representation of a particles spin, the additional phase-free discrete variable discussed by Wigner [73]. The consideration by Y. S. Kim *et al.*, see ref. [36a, formula (14)], ensures that the problem of physical significance of Evans-Vigier-type longitudinal modes is related with the problem of the normalization and of existence of the mass of a particle transformed on the $(1, 0) \oplus (0, 1)$ representation of the Poincaré group. Considering explicit forms of the $(1, 0) \oplus (0, 1)$ “bispinors” in the light-front formulation [20] of the quantum field theory of this representation (the Weinberg-Soper formalism) D. V. Ahluwalia and M. Sawicki showed that in the massless limit one has only two non-vanishing Dirac-like solutions. The “bispinor” corresponding to the longitudinal solution is directly proportional to the mass of the particle. So, the massless limit of this theory, the relevance of the $E(2)$ group to describing physical phenomena and the problem of what is mass deserve further consideration.

Antisymmetric tensor fields. To my knowledge researches of antisymmetric tensor fields in the quantum theory began from the paper by V. I. Ogievetskiĭ and I. V. Polubarinov [56]. They claimed that the antisymmetric tensor field (*notoph* in the terminology used, which I find quite suitable) can be longitudinal in the quantum theory, owing to the new gauge invariance

$$F_{\mu\nu} \rightarrow F_{\mu\nu} + \partial_\mu \Lambda - \partial_\nu \Lambda \quad (24)$$

and applications of the supplementary conditions. The result by Ogievetskiĭ and Polubarinov has been repeated by K. Hayashi [37], M. Kalb and P. Ramond [45] and T. E. Clark *et al.* [18]. The Lagrangian ($F_k = i\epsilon_{kjmn}F_{jm,n}$)

$$\mathcal{L}^H = \frac{1}{8}F_k F_k = -\frac{1}{4}(\partial_\mu F_{\nu\alpha})(\partial_\mu F_{\nu\alpha}) + \frac{1}{2}(\partial_\mu F_{\nu\alpha})(\partial_\nu F_{\mu\alpha}) \quad (25)$$

after the application of the Fermi method *mutatis mutandis* (comparing with the case of the 4-vector potential field) yields the spin dynamical invariant to be equal to zero. While several authors insisted on the transversality of the antisymmetric tensor field and the necessity of gauge-independent consideration [15,67,14] perpetually this interpretation (‘longitudinity’) has become wide-accepted. In refs. [6,7] an antisymmetric tensor *matter* field was studied and it appears to be also longitudinal, but to have two degrees of freedom. Unfortunately, the authors of the cited work regarded only a massless *real* field and did not take into account the physical reality of the dual field corresponding to an antiparticle. But, what is important, L. Avdeev and M. Chizhov noted [7] that in such a framework there exist δ' transversal solutions, which cannot be interpreted as relativistic particles.

If an antisymmetric tensor field would be pure longitudinal, it appears failure to understand, why in the classical electromagnetism we are convinced that an antisymmetric tensor field is transversal. Does this signifies one must abandon the Correspondence Principle? Moreover, this result contradicts with the Weinberg theorem $B - A = \lambda$, ref. [71b]. This situation has been later analyzed in refs. [23,33,25,28,29] and it was found that indeed the “longitudinal nature” of antisymmetric tensor fields is connected with the application of the generalized Lorentz condition to the quantum states: $\partial_\mu F^{\mu\nu}|\Psi\rangle = 0$. Such a procedure leads also (like in the case of the treatment of the 4-vector potential field without proper regarding the phase field) to the problem of the indefinite metric which was noted by Gupta and Bleuler. So, it is already obviously from methodological viewpoints that the grounds for regarding only particular cases can be doubted by the Lorentz symmetry principles. Ignoring the phase field of Dirac-Fock-Podol’sky-Staruszkiewicz or ignoring χ functions [27]

related with the 4-current and, hence, with the non-zero value of $\partial_\mu F^{\mu\nu}$ can put obstacles in the way of creation of the unified field theory and embarrass understanding the physical content dictated by the Relativity Theory.

II. THE WEINBERG FORMALISM

In the beginning of the sixties the $2(2j + 1)$ - component approach has been proposed in order to construct a Lorentz-invariant interaction S -matrix from the first principles [8,43,71,70,72,51,69]. The authors had thus some hopes on an adequate perturbation calculus for processes including higher-spin particles which appeared in disposition of physicists in that time. The field theory in that time was in some troubles.

The Weinberg *ansatz* for the $(j, 0) \oplus (0, j)$ field theory are simple and obvious [71a,p.B1318]:

a) relativistic invariance

$$U[\Lambda, a]\psi_n(x)U^{-1}[\Lambda, a] = \sum_m D_{nm}[\Lambda^{-1}]\psi_m(\Lambda x + a) \quad , \quad (26)$$

where $D_{nm}[\Lambda]$ is the corresponding representation of Λ ;

b) causality

$$[\psi_n(x), \psi_m(y)]_\pm = 0 \quad (27)$$

for $(x - y)$ spacelike, which guarantees the commutator of the Hamiltonian density $[\mathcal{H}(x), \mathcal{H}(y)] = 0$, provided that $\mathcal{H}(x)$ contains an even number of fermion field factors. The interaction Hamiltonian $\mathcal{H}(x)$ is constructed out of the creation and annihilation operators for the free particles described by some H_0 , the free-particle part of the Hamiltonian. Thus, the $(j, 0) \oplus (0, j)$ field

$$\psi(x) = \begin{pmatrix} \varphi(x) \\ \chi(x) \end{pmatrix} \quad (28)$$

transforms according to (26), where

$$\mathcal{D}^{(j)}[\Lambda] = \begin{pmatrix} D^{(j)}[\Lambda] & 0 \\ 0 & \overline{D}^{(j)}[\Lambda] \end{pmatrix}, \quad D^{(j)}[\Lambda] = \overline{D}^{(j)}[\Lambda^{-1}]^\dagger \quad , \quad \mathcal{D}^{(j)}[\Lambda]^\dagger = \beta \mathcal{D}^{(j)}[\Lambda^{-1}] \beta \quad , \quad (29)$$

with

$$\beta = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \quad , \quad (30)$$

and, hence, for pure Lorentz transformations (boosts)

$$D^{(j)}[L(\mathbf{p})] = \exp(-\hat{p} \cdot \mathbf{J}^{(j)}\theta) \quad , \quad (31a)$$

$$\overline{D}^{(j)}[L(\mathbf{p})] = \exp(+\hat{p} \cdot \mathbf{J}^{(j)}\theta) \quad , \quad (31b)$$

with $\sinh \theta \equiv |\mathbf{p}|/m$. Dynamical equations, which Weinberg proposed, are (Eqs. (7.17) and (7.18) of the first paper [71]):

$$\bar{\Pi}(-i\partial)\varphi(x) = m^{2j}\chi(x) \quad , \quad (32a)$$

$$\Pi(-i\partial)\chi(x) = m^{2j}\varphi(x) \quad . \quad (32b)$$

They are rewritten into the form (Eq. (7.19) of [71a])

$$\left[\gamma^{\mu_1\mu_2\ldots\mu_{2j}} \partial_{\mu_1} \partial_{\mu_2} \ldots \partial_{\mu_{2j}} + m^{2j} \right] \psi(x) = 0 \quad , \quad (33)$$

with the Barut-Muzinich-Williams matrices [8]

$$\gamma^{\mu_1\mu_2\ldots\mu_{2j}} = -i^{2j} \begin{pmatrix} 0 & t^{\mu_1\mu_2\ldots\mu_{2j}} \\ \bar{t}^{\mu_1\mu_2\ldots\mu_{2j}} & 0 \end{pmatrix} \quad . \quad (34)$$

The following notation was used

$$\Pi_{\sigma'\sigma}^{(j)}(q) \equiv (-1)^{2j} t_{\sigma'\sigma}^{\mu_1\mu_2\ldots\mu_{2j}} q_{\mu_1} q_{\mu_2} \ldots q_{\mu_{2j}} \quad , \quad (35)$$

$$\bar{\Pi}_{\sigma'\sigma}^{(j)}(q) \equiv (-1)^{2j} \bar{t}_{\sigma'\sigma}^{\mu_1\mu_2\ldots\mu_{2j}} q_{\mu_1} q_{\mu_2} \ldots q_{\mu_{2j}} \quad , \quad \bar{\Pi}^{(j)*}(q) = C \Pi^{(j)} C^{-1} \quad , \quad (36)$$

with C being the matrix of the charge conjugation in the $2j+1$ - dimension representation. The tensor t is defined in a following manner:

- $t_{\sigma'\sigma}^{\mu_1\mu_2\ldots\mu_{2j}}$ is a $2j+1$ matrix with $\sigma, \sigma' = j, j-1, \ldots, -j$; $\mu_1, \mu_2 \ldots \mu_{2j} = 0, 1, 2, 3$;
- t is symmetric in all μ 's;
- t is traceless in all μ 's, *i.e.*, $g_{\mu_1\mu_2} t_{\sigma'\sigma}^{\mu_1\mu_2\ldots\mu_{2j}}$ and with all permutations of upper indices;
- t is a tensor under Lorentz transformations,

$$D^{(j)}[\Lambda] t^{\mu_1\mu_2\ldots\mu_{2j}} D^{(j)}[\Lambda]^\dagger = \Lambda_{\nu_1}^{\mu_1} \Lambda_{\nu_2}^{\mu_2} \ldots \Lambda_{\nu_{2j}}^{\mu_{2j}} t^{\nu_1\nu_2\ldots\nu_{2j}} \quad , \quad (37a)$$

$$\bar{D}^{(j)}[\Lambda] \bar{t}^{\mu_1\mu_2\ldots\mu_{2j}} \bar{D}^{(j)}[\Lambda]^\dagger = \Lambda_{\nu_1}^{\mu_1} \Lambda_{\nu_2}^{\mu_2} \ldots \Lambda_{\nu_{2j}}^{\mu_{2j}} \bar{t}^{\nu_1\nu_2\ldots\nu_{2j}} \quad . \quad (37b)$$

For instance, in the $j=1$ case $t^{00} = \mathbb{1}$, $t^{0i} = t^{i0} = J_i$ and $t^{ij} = \{J_i, J_j\} - \delta_{ij}$, with J_i being the $j=1$ spin matrices and the metric $g_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ being used. Furthermore, for this representation

$$\bar{t}^{\mu_1\mu_2\ldots\mu_{2j}} = \pm t^{\mu_1\mu_2\ldots\mu_{2j}} \quad , \quad (38)$$

the sign being $+1$ or -1 according to whether the μ 's contain altogether an even or an odd number of space-like indices.

The Feynman diagram technique has been built and some properties with respect to discrete symmetry operations have been studied. The propagator used in the Feynman diagram technique is found not to be the propagator arising from the Wick theorem because of extra terms proportional to equal-time δ functions and their derivatives appearing if one uses the time-ordering product of field operators $\langle T \{ \psi_\alpha(x) \bar{\psi}_\beta(y) \} \rangle_0$. The covariant propagator is defined by

$$S_{\alpha\beta}(x-y) = (2\pi)^{-3} m^{-2j} M_{\alpha\beta}(-i\partial) \int \frac{d^3\mathbf{p}}{2\omega(\mathbf{p})} [\theta(x-y) \exp\{ip \cdot (x-y)\} + \theta(y-x) \exp\{ip \cdot (y-x)\}] = -im^{-2j} M_{\alpha\beta}(-i\partial) \Delta^C(x-y) \quad , \quad (39)$$

where

$$M(p) = \begin{pmatrix} m^{2j} & \Pi(p) \\ \bar{\Pi}(p) & m^{2j} \end{pmatrix} \quad , \quad (40)$$

and $\Delta^C(x)$ is the covariant $j = 0$ propagator.

For massless particles the Weinberg theorem about connections between the helicity of a particle and the representation of the group (A, B) which the corresponding field transforms on has been proved. It says: “A massless particle operator $a(\mathbf{p}, \lambda)$ of helicity λ can only be used to construct fields which transform according to representations (A, B) , such that $B - A = \lambda$. For instance, a left-circularly polarized photon with $\lambda = -1$ can be associated with $(1, 0)$, $(\frac{3}{2}, \frac{1}{2})$, $(2, 1)$... fields, but *not* with the vector potential, $(\frac{1}{2}, \frac{1}{2})$... [It is not the case of a massive particle.] A field can be constructed out of $2j + 1$ operators $a(\mathbf{p}, \sigma)$ for any representation (A, B) that “contains” j , such that $j = A + B$ or $A + B - 1$... or $|A - B|$, [e. g., a $j = 1$ particle] field could be a four-vector $(\frac{1}{2}, \frac{1}{2})$... [*i.e.*, built out of the vector potential].”

In subsequent papers Weinberg showed that it is possible to construct fields transformed on other representations of the Lorentz group but unlikely can be considered as fundamental ones. The prescription for constructing fields have been given in ref. [71c,p.1895]. “Any irreducible field $\psi^{(A,B)}$ for a particle of spin j may be constructed by applying a suitable differential operator of order $2B$ to the field $\psi^{(j,0)}$, provided that A, B , and j satisfy the triangle inequality $|A - B| \leq j \leq A + B$.” For example, from the self-dual antisymmetric tensor $F^{\mu\nu}$ the $(1/2, 1/2)$ field $\partial_\mu F^{\mu\nu}$, the $(0, 1)$ field $\epsilon_{\mu\nu\lambda\rho} \partial^\lambda \partial_\sigma F^{\rho\sigma}$ have been constructed. Moreover, various invariant-type interactions have been tabulated [71b,p.B890] and [71c,Section III]. While one can also use fields from different representations of the Lorentz group to obtain some physical predictions, in my opinion, such a wisdom could lead us to certain mathematical inconsistencies (like the indefinite metric problem and the subtraction of infinities [19]). The applicability of the procedure mentioned above to massless states should still be analyzed in detail.

Finally, we would like to cite a few paragraphs from other Weinberg’s works. In the paper of 1965 Weinberg proposed his concept how to deal with several puzzles noted before [72c]: “... Tensor fields cannot by themselves be used to construct the interaction $H'(t)$, because the coefficients of the operators for creation or annihilation of particles of momentum p and spin j would vanish as p^j for $p \rightarrow 0$, in contradiction with the known existence of inverse-square-law forces. We are therefore forced to turn from these tensor fields to the potentials... The potentials are not tensor fields; indeed, they cannot be, for we know from a very general theorem [71b] that no symmetric tensor field of rank j can be constructed from the creation and annihilation operators of massless particles of spin j . It is for this reason that some field theorists have been led to introduce fictitious photons and gravitons of helicity other than $\pm j$, as well as the indefinite metric that must accompany them. Preferring to avoid such unphysical monstrosities, we must ask now what sort of coupling we can give our nontensor potentials without losing the Lorentz invariance of the S matrix?... Those in which the potential is coupled to a conserved current.” Thus, gauge models obtain some physical grounds from the Lorentz invariance.

III. THE WEINBERG FORMALISM IN NEW DEVELOPMENT

In the papers [62] another equation in the $(1, 0) \oplus (0, 1)$ representation has been proposed. It reads (p_μ is the differential operator, $E = \sqrt{\mathbf{p}^2 + m^2}$)

$$\left[\gamma_{\mu\nu} p_\mu p_\nu + \frac{i(\partial/\partial t)}{E} m^2 \right] \psi = 0 \quad . \quad (41)$$

The auxiliary condition

$$(p_\mu p_\mu + m^2) \psi = 0 \quad (42)$$

is implied. "...In the momentum representation the wave equation may be written as [62b, formula (12)]

$$[\pm \gamma_{\mu\nu} p_\mu p_\nu + m^2] U_\pm(\mathbf{p}) = 0 \quad . \quad (43)$$

corresponding to the particle and antiparticle with the column vector $U_+(\mathbf{p})$ and $U_-(\mathbf{p})$, respectively." Many dynamical features of this approach have been analyzed in those papers but, unfortunately, the author erroneously claimed that the two formulation (the Weinberg's one and his own formulation) "are equivalent in physical content". The matters related with the discrete symmetry operations have been analyzed in detail in the recent years only by Ahluwalia *et al.*, ref. [3,5] and [4b]. In this Section first of all let us follow the arguments of the cited papers. According to the Wigner rules (29,31a,31b) in the notation of papers [3] one has

$$\phi_R(\mathbf{p}) = \Lambda_R \phi_R(\mathbf{0}) = \exp(+\mathbf{J} \cdot \boldsymbol{\varphi}) \phi_R(\mathbf{0}) \quad , \quad (44a)$$

$$\phi_L(\mathbf{p}) = \Lambda_L \phi_L(\mathbf{0}) = \exp(-\mathbf{J} \cdot \boldsymbol{\varphi}) \phi_L(\mathbf{0}) \quad , \quad (44b)$$

with $\phi_{R,L}(\mathbf{p})$ being $(j, 0)$ right- and $(0, j)$ left- 'bispinors' in the momentum representation, respectively; $\boldsymbol{\varphi}$ are the parameters of the Lorentz boost. By means of the explicit application (44a,44b) the $u_\sigma(\mathbf{p})$ and $v_\sigma(\mathbf{p})$ bispinors have been found in the $j = 1$, $j = 3/2$ and $j = 2$ cases [3a] in the generalized canonical representation. For the $(1, 0) \oplus (0, 1)$ 'bispinors' see, *e.g.*, formulas (7) of [3b]. It was proved that the states answering for positive- and negative- energy solutions have intrinsic parities $+1$ and -1 , respectively, when applying the space inversion operation. The conclusion has been achieved in the Fock secondary-quantization space too, thus proving that we have an explicit example of the theory envisaged by Bargmann, Wightman and Wigner (BWW) long ago, ref. [73b]. The remarkable feature of this formulation is: a boson and its antiboson have opposite intrinsic parities. Origins of this fact have been explained in ref. [2] thanks to an anonymous referee. Namely, "the relative phase ϵ between particle and antiparticle states is not arbitrary and is naturally defined as:

$$U(P)U(C) = \epsilon U(C)U(P) \quad . \quad (45)$$

[One can prove that] $\epsilon = \pm 1$ by using associativity of the group law." Depending on the operations of the space inversion and of the charge conjugation either commute or anticommute we obtain either the same or the opposite values of parities for particle and antiparticle. The commutator of these operations in the Fock space is "a function of the Charge operator with formal and phenomenological consequences", refs. [55,5,2]. The massless limit of the theory in the $(1, 0) \oplus (0, 1)$ representation was studied in [1,2,4] with the following

result achieved (cited from [2]): “Present theoretical arguments suggest that in strong fields, or high-frequency phenomenon, Maxwell equations may not be an adequate description of nature. Whether this is so can only be decided by experiment(s). Similar conclusions, in apparently very different framework, have been independently arrived at by M. Evans and communicated to the author.”

In ref. [4] the properties of the light-front-form $(1/2, 0) \oplus (0, 1/2)$ bispinors have been under study. An unexpected result has been obtained that they do not get interchanged under the operation of parity. Thus, one must take into account the evolution of a physical system not only along x^+ but also along the x^- direction. In [5] the Majorana-McLennan-Case construct has been analyzed and interesting mathematical and phenomenological connections have been found. The analysis resulted in a serie of papers of both mine and others in many physical journals, but a detailed presentation of these ideas is out of a subject of this paper.⁶

In a recent serie of my papers [24–29] I slightly went from this BWV-type theory and advocated co-existence of two Weinberg’s equations with opposite signs in the mass term for the spin $j = 1$ case. Their connections with classical and quantum electrodynamics, and (the paper in preparation) the possibility of the use of the same $(j, 0) \oplus (0, j)$ field operator to obtain the Majorana states or the Dirac states were studied. The reason for this reformulation is that the Weinberg equations are of the second order in derivatives and each of them provides dispersional relations with both positive and negative signs of the energy.⁷ Below I present a brief content of those papers.

The $2(2j+1)$ - component analogues of the Dirac functions in the momentum space were earlier defined as

$$\mathcal{U}(\mathbf{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} D^J(\alpha(\mathbf{p})) \xi_\sigma \\ D^J(\alpha^{-1\dagger}(\mathbf{p})) \xi_\sigma \end{pmatrix} , \quad (47)$$

for positive-energy states, and

⁶I still present references of my work related with these problems. Here they are: V. V. Dvoeglazov, Nuovo Cim. A108 (1995) 1467; ibid 111B (1996) 483; Int. J. Theor. Phys. 34 (1995) 2467; ibid 35 (1996) 115; Rev. Mex. Fis. Suppl. 41 (1995) 159; Hadronic J. Suppl. 10 (1995) 349; ibid 10 (1995) 359; Bol. Soc. Mex. Fis. 9-3 (1995) 28; Proc. ICTE’95, Cuautitlán, Sept. 1995; Proc. IV Wigner Symp., Guadalajara, Aug. 1995; Inv. Científica, in press. Several papers are now under review in various journals.

⁷Generally speaking, the both massive Weinberg’s equations possess tachyonic solutions. While this content is *not* already in a strong contradiction with experimental observations (see the enumeration of experiments on superluminal phenomena above and the papers of E. Recami) someone can still regard this as a shortcoming before that time when they would be given adequate explanation. We note that one can still escape from this problem by choosing the particular a and b in the equation below:

$$\left[\gamma_{\alpha\beta} p_\alpha p_\beta + a p_\alpha p_\alpha + b m^2 \right] \psi = 0 \quad . \quad (46)$$

Thus, one can obtain the Hammer-Tucker equations [69], which have dispersion relations $E = \pm \sqrt{\mathbf{p}^2 + m^2}$ if one restricts by particles with mass.

$$\mathcal{V}(\mathbf{p}) = \frac{m}{\sqrt{2}} \left(\begin{array}{c} D^J \left(\alpha(\mathbf{p}) \Theta_{[1/2]} \right) \xi_\sigma^* \\ D^J \left(\alpha^{-1\dagger}(\mathbf{p}) \Theta_{[1/2]} \right) (-1)^{2J} \xi_\sigma^* \end{array} \right) , \quad (48)$$

for negative-energy states, *e.g.*, ref. [54]. The following notation was used

$$\alpha(\mathbf{p}) = \frac{p_0 + m + (\boldsymbol{\sigma} \cdot \mathbf{p})}{\sqrt{2m(p_0 + m)}}, \quad \Theta_{[1/2]} = -i\sigma_2 \quad . \quad (49)$$

For example, in the case of spin $j = 1$, one has

$$D^1(\alpha(\mathbf{p})) = 1 + \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(p_0 + m)} , \quad (50a)$$

$$D^1(\alpha^{-1\dagger}(\mathbf{p})) = 1 - \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(p_0 + m)} , \quad (50b)$$

$$D^1(\alpha(\mathbf{p})\Theta_{[1/2]}) = \left[1 + \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(p_0 + m)} \right] \Theta_{[1]} , \quad (50c)$$

$$D^1(\alpha^{-1\dagger}(\mathbf{p})\Theta_{[1/2]}) = \left[1 - \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(p_0 + m)} \right] \Theta_{[1]} ; \quad (50d)$$

$\Theta_{[1/2]}, \Theta_{[1]}$ are the Wigner time-reversal operators for spin 1/2 and 1, respectively. These definitions lead to the formulation in which the physical content given by positive and negative-energy ‘bispinors’ is the same (like in the paper of R. H. Tucker and C. L. Hammer [69]). One can consider that $\mathcal{V}_\sigma(\mathbf{p}) = (-1)^{1-\sigma} \gamma_5 S_{[1]}^c \mathcal{U}_{-\sigma}(\mathbf{p})$ and, thus, the explicit form of the negative-energy solutions would be the same as of the positive-energy solutions in accordance with definitions (47,48).

Next, let me look at the Proca equations for a $j = 1$ massive particle

$$\partial_\mu F_{\mu\nu} = m^2 A_\nu , \quad (51a)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (51b)$$

in the form given by refs. [62,47]. The Euclidean metric, $x_\mu = (\vec{x}, x_4 = it)$ and notation $\partial_\mu = (\vec{\nabla}, -i\partial/\partial t)$, $\partial_\mu^2 = \vec{\nabla}^2 - \partial_t^2$, are used. By means of the choice of $F_{\mu\nu}$ components as physical variables one can rewrite the set of equations to

$$m^2 F_{\mu\nu} = \partial_\mu \partial_\alpha F_{\alpha\nu} - \partial_\nu \partial_\alpha F_{\alpha\mu} \quad (52)$$

and

$$\partial_\lambda^2 F_{\mu\nu} = m^2 F_{\mu\nu} . \quad (53)$$

It is easy to show that they can be represented in the form ($F_{44} = 0$, $F_{4i} = iE_i$ and $F_{jk} = \epsilon_{jki} B_i$; $p_\alpha = -i\partial_\alpha$):

$$\begin{cases} (m^2 + p_4^2) E_i + p_i p_j E_j + i\epsilon_{ijk} p_4 p_j B_k = 0 \\ (m^2 + \vec{p}^2) B_i - p_i p_j B_j + i\epsilon_{ijk} p_4 p_j E_k = 0 \end{cases} , \quad (54)$$

or

$$\begin{cases} [m^2 + p_4^2 + \mathbf{p}^2 - (\mathbf{J} \cdot \mathbf{p})^2]_{ij} E_j + p_4(\mathbf{J} \cdot \mathbf{p})_{ij} B_j = 0 \\ [m^2 + (\mathbf{J} \cdot \mathbf{p})^2]_{ij} B_j + p_4(\mathbf{J} \cdot \mathbf{p})_{ij} E_j = 0 \end{cases} \quad (55)$$

After adding and subtracting the obtained equations yield

$$\begin{cases} m^2(\mathbf{E} + i\mathbf{B})_i + p_\alpha p_\alpha \mathbf{E}_i - (\mathbf{J} \cdot \mathbf{p})_{ij}^2 (\mathbf{E} - i\mathbf{B})_j + p_4(\mathbf{J} \cdot \mathbf{p})_{ij} (\mathbf{B} + i\mathbf{E})_j = 0 \\ m^2(\mathbf{E} - i\mathbf{B})_i + p_\alpha p_\alpha \mathbf{E}_i - (\mathbf{J} \cdot \mathbf{p})_{ij}^2 (\mathbf{E} + i\mathbf{B})_j + p_4(\mathbf{J} \cdot \mathbf{p})_{ij} (\mathbf{B} - i\mathbf{E})_j = 0 \end{cases} \quad (56)$$

with $(\mathbf{J}_i)_{jk} = -i\epsilon_{ijk}$ being the $j = 1$ spin matrices. Equations are equivalent (within a constant factor) to the Hammer-Tucker equation [69]

$$(\gamma_{\alpha\beta} p_\alpha p_\beta + p_\alpha p_\alpha + 2m^2)\psi_1 = 0 \quad (57)$$

in the case of the choice $\chi = \mathbf{E} + i\mathbf{B}$ and $\varphi = \mathbf{E} - i\mathbf{B}$, $\psi_1 = \text{column}(\chi, \varphi)$. Matrices $\gamma_{\alpha\beta}$ are the covariantly defined matrices of Barut, Muzinich and Williams [8] for spin $j = 1$. The equation (57) for massive particles is characterized by positive- and negative-energy solutions with a physical dispersion only $E_p = \pm\sqrt{\mathbf{p}^2 + m^2}$, the determinant is equal to

$$\text{Det} [\gamma_{\alpha\beta} p_\alpha p_\beta + p_\alpha p_\alpha + 2m^2] = -64m^6(p_0^2 - \mathbf{p}^2 - m^2)^3 \quad (58)$$

However, there is another equation which also do not have ‘acausal’ solutions. The second one (with $a = -1$ and $b = -2$, see (46)) is

$$(\gamma_{\alpha\beta} p_\alpha p_\beta - p_\alpha p_\alpha - 2m^2)\psi_2 = 0 \quad (59)$$

In the tensor form it leads to the equations which are dual to (54)

$$\begin{cases} (m^2 + \mathbf{p}^2)C_i - p_i p_j C_j - i\epsilon_{ijk} p_4 p_j D_k = 0 \\ (m^2 + p_4^2)D_i + p_i p_j D_j - i\epsilon_{ijk} p_4 p_j C_k = 0 \end{cases} \quad (60)$$

They can be rewritten in the form, *cf.* (52),

$$m^2 \tilde{F}_{\mu\nu} = \partial_\mu \partial_\alpha \tilde{F}_{\alpha\nu} - \partial_\nu \partial_\alpha \tilde{F}_{\alpha\mu} \quad (61)$$

with $\tilde{F}_{4i} = iD_i$ and $\tilde{F}_{jk} = -\epsilon_{jki} C_i$. The vector C_i is an analog of E_i and D_i is an analog of B_i because in some cases it is convenient to equate $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$, $\epsilon_{1234} = -i$. The following properties of the antisymmetric Levi-Civita tensor

$$\epsilon_{ijk}\epsilon_{ijl} = 2\delta_{kl} \quad , \quad \epsilon_{ijk}\epsilon_{ilm} = (\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}) \quad ,$$

and

$$\epsilon_{ijk}\epsilon_{lmn} = \text{Det} \begin{pmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{pmatrix}$$

have been used.

Comparing the structure of the Weinberg equation ($a = 0$, $b = 1$) with the Hammer-Tucker ‘doubles’ one can convince ourselves that the former can be represented in the tensor form:

$$m^2 F_{\mu\nu} = \partial_\mu \partial_\alpha F_{\alpha\nu} - \partial_\nu \partial_\alpha F_{\alpha\mu} + \frac{1}{2}(m^2 - \partial_\lambda^2) F_{\mu\nu} \quad , \quad (62)$$

that corresponds to Eq. (64a). However, as we learned, it is possible to build an equation — ‘double’ :

$$m^2 \tilde{F}_{\mu\nu} = \partial_\mu \partial_\alpha \tilde{F}_{\alpha\nu} - \partial_\nu \partial_\alpha \tilde{F}_{\alpha\mu} + \frac{1}{2}(m^2 - \partial_\lambda^2) \tilde{F}_{\mu\nu} \quad , \quad (63)$$

that corresponds to Eq. (64b). The Weinberg’s set of equations is written in the form:

$$(\gamma_{\alpha\beta} p_\alpha p_\beta + m^2) \psi_1 = 0 \quad , \quad (64a)$$

$$(\gamma_{\alpha\beta} p_\alpha p_\beta - m^2) \psi_2 = 0 \quad . \quad (64b)$$

Thanks to the Klein-Gordon equation (53) these equations are equivalent to the Proca tensor equations (52,61), and to the Hammer-Tucker ‘doubles’, in a free case. However, if an interaction is included, one cannot say that. The second equation (64b) coincides with the Ahluwalia *et al.* equation for v spinors (Eq. (16) of ref. [3b]) or with Eq. (12) of ref. [62b]. Thus, the general solution describing $j = 1$ states can be presented as a superposition

$$\Psi^{(1)} = c_1 \psi_1^{(1)} + c_2 \psi_2^{(1)} \quad , \quad (65)$$

where the constants c_1 and c_2 are to be defined from the boundary, initial and normalization conditions. Let me note a surprising fact: while both the massive Proca equations (or the Hammer-Tucker ones) and the Klein-Gordon equation do not possess ‘non-physical’ solutions, their sum, Eqs. (62,63), or the Weinberg equations (64a,64b), acquire tachyonic solutions. Next, equations (64a) and (64b) can recast in another form (index “ T ” denotes a transpose matrix):

$$\left[\gamma_{44} p_4^2 + 2\gamma_{4i}^T p_4 p_i + \gamma_{ij} p_i p_j - m^2 \right] \psi_1^{(2)} = 0 \quad , \quad (66a)$$

$$\left[\gamma_{44} p_4^2 + 2\gamma_{4i}^T p_4 p_i + \gamma_{ij} p_i p_j + m^2 \right] \psi_2^{(2)} = 0 \quad , \quad (66b)$$

respectively, if understand $\psi_1^{(2)} \sim \text{column}(B_i + iE_i, \quad B_i - iE_i) = i\gamma_5 \gamma_{44} \psi_1^{(1)}$ and $\psi_2^{(2)} \sim \text{column}(D_i + iC_i, \quad D_i - iC_i) = i\gamma_5 \gamma_{44} \psi_2^{(1)}$. The general solution is again a linear combination

$$\Psi^{(2)} = c_1 \psi_1^{(2)} + c_2 \psi_2^{(2)} \quad . \quad (67)$$

From, *e.g.*, Eq. (64a), dividing $\psi_1^{(1)}$ into longitudinal and transversal parts one can come to the equations

$$\begin{aligned} & \left[E^2 - \vec{p}^2 \right] (\mathbf{E} + i\mathbf{B})^\parallel - m^2 (\mathbf{E} - i\mathbf{B})^\parallel + \\ & + \left[E^2 + \vec{p}^2 - 2E(\mathbf{J} \cdot \mathbf{p}) \right] (\mathbf{E} + i\mathbf{B})^\perp - m^2 (\mathbf{E} - i\mathbf{B})^\perp = 0 \quad , \end{aligned} \quad (68)$$

and

$$\begin{aligned} & \left[E^2 - \vec{p}^2 \right] (\mathbf{E} - i\mathbf{B})^\parallel - m^2 (\mathbf{E} + i\mathbf{B})^\parallel + \\ & + \left[E^2 + \vec{p}^2 + 2E(\mathbf{J} \cdot \mathbf{p}) \right] (\mathbf{E} - i\mathbf{B})^\perp - m^2 (\mathbf{E} + i\mathbf{B})^\perp = 0 \quad . \end{aligned} \quad (69)$$

One can see that in the classical field theory antisymmetric tensor matter fields are the fields having transversal components in the massless limit. Under the transformations $\psi_1^{(1)} \rightarrow \gamma_5 \psi_2^{(1)}$ or $\psi_1^{(2)} \rightarrow \gamma_5 \psi_2^{(2)}$ the set of equations (64a) and (64b), or Eqs. (66a) and (66b), leave to be invariant. The origin of this fact is the dual invariance of the set of the Proca equations. In the matrix form dual transformations correspond to the chiral transformations.

Let me consider the question of the ‘double’ solutions on the basis of spinorial analysis. In ref. [62a,p.1305] (see also [12, p.60-61]) relations between the Weinberg $j = 1$ “bispinor” (bivector, indeed) and symmetric spinors of $2j$ - rank have been discussed. It was noted there: “The wave function may be written in terms of two three-component functions $\psi = \text{column}(\chi \quad \varphi)$, that, for the continuous group, transform independently each of other and that are related to two symmetric spinors:

$$\chi_1 = \chi_{ii}, \quad \chi_2 = \sqrt{2}\chi_{i\dot{2}}, \quad \chi_3 = \chi_{\dot{2}\dot{2}} \quad , \quad (70a)$$

$$\varphi_1 = \varphi^{11}, \quad \varphi_2 = \sqrt{2}\varphi^{12}, \quad \varphi_3 = \varphi^{22} \quad , \quad (70b)$$

when the standard representation for the spin-one matrices, with J_3 diagonal is used.” Under the inversion operation we have the following rules [12, p.59]: $\varphi^\alpha \rightarrow \chi_{\dot{\alpha}}$, $\chi_{\dot{\alpha}} \rightarrow \varphi^\alpha$, $\varphi_\alpha \rightarrow -\chi^{\dot{\alpha}}$ and $\chi^{\dot{\alpha}} \rightarrow -\varphi_\alpha$. Hence, one can deduce (if one understand $\chi_{\dot{\alpha}\dot{\beta}} = \chi_{\{\dot{\alpha}\dot{\beta}\}}$, $\varphi^{\alpha\beta} = \varphi^{\{\alpha\beta\}}$)

$$\chi_{ii} \rightarrow \varphi^{11} \quad , \quad \chi_{\dot{2}\dot{2}} \rightarrow \varphi^{22} \quad , \quad \chi_{\{i\dot{2}\}} \rightarrow \varphi^{\{12\}} \quad , \quad (71a)$$

$$\varphi^{11} \rightarrow \chi_{ii} \quad , \quad \varphi^{22} \rightarrow \chi_{\dot{2}\dot{2}} \quad , \quad \varphi^{\{12\}} \rightarrow \chi_{\{i\dot{2}\}} \quad . \quad (71b)$$

However, this definition of symmetric spinors of the second rank χ and φ is ambiguous. We are also able to define, *e.g.*, $\tilde{\chi}_{\dot{\alpha}\dot{\beta}} = \chi_{\{\dot{\alpha}\dot{\beta}\}} H_{\dot{\beta}}$ and $\tilde{\varphi}^{\alpha\beta} = \varphi^{\{\alpha\beta\}} \Phi^\beta$, where $H_{\dot{\beta}} = \varphi_{\dot{\beta}}^*$, $\Phi^\beta = (\chi^{\dot{\beta}})^*$. It is straightforwardly showed that in the framework of the second definition we have under the space-inversion operation:

$$\tilde{\chi}_{ii} \rightarrow -\tilde{\varphi}^{11} \quad , \quad \tilde{\chi}_{\dot{2}\dot{2}} \rightarrow -\tilde{\varphi}^{22} \quad , \quad \tilde{\chi}_{\{i\dot{2}\}} \rightarrow -\tilde{\varphi}^{\{12\}} \quad , \quad (72a)$$

$$\varphi^{11} \rightarrow -\tilde{\chi}_{ii} \quad , \quad \varphi^{22} \rightarrow -\tilde{\chi}_{\dot{2}\dot{2}} \quad , \quad \varphi^{\{12\}} \rightarrow -\tilde{\chi}_{\{i\dot{2}\}} \quad . \quad (72b)$$

The Weinberg ‘bispinor’ $(\chi_{\dot{\alpha}\dot{\beta}} \quad \varphi^{\alpha\beta})$ corresponds to the equations (66a) and (66b), meanwhile $(\tilde{\chi}_{\dot{\alpha}\dot{\beta}} \quad \tilde{\varphi}^{\alpha\beta})$, to the equation (64a) and (64b). Similar conclusions can be arrived at in the case of the parity definition as $P^2 = -1$. Transformation rules are then $\varphi^\alpha \rightarrow i\chi_{\dot{\alpha}}$, $\chi_{\dot{\alpha}} \rightarrow i\varphi^\alpha$, $\varphi_\alpha \rightarrow -i\chi^{\dot{\alpha}}$ and $\chi^{\dot{\alpha}} \rightarrow -i\varphi_\alpha$, ref. [12, p.59]. Hence, $\chi_{\dot{\alpha}\dot{\beta}} \leftrightarrow -\varphi^{\alpha\beta}$ and $\tilde{\chi}_{\dot{\alpha}\dot{\beta}} \leftrightarrow -\tilde{\varphi}^{\alpha\beta}$, but $\varphi^\alpha{}_\beta \leftrightarrow \chi_{\dot{\alpha}}{}^{\dot{\beta}}$ and $\tilde{\varphi}^\alpha{}_\beta \leftrightarrow \tilde{\chi}_{\dot{\alpha}}{}^{\dot{\beta}}$.

In order to consider the corresponding dynamical content we should choose an appropriate Lagrangian. In the framework of this essay we concern with the Lagrangian which is similar to the one used in earlier works on the $2(2j+1)$ formalism (see for references [23]). Our Lagrangian includes additional terms which respond to the Weinberg ‘double’ and does not suffer from the problems noted in the old works. Here it is:^{8, 9}

⁸Under field functions we assume $\psi_{1,2}^{(1)}$. Of course, one can use another form with substitutions: $\psi_{1,2}^{(1)} \rightarrow \psi_{2,1}^{(2)}$ and $\gamma_{\mu\nu} \rightarrow \tilde{\gamma}_{\mu\nu}$, where $\tilde{\gamma}_{\mu\nu} \equiv \gamma_{\mu\nu}^T \equiv \gamma_{44}\gamma_{\mu\nu}\gamma_{44}$.

⁹Questions related with other possible Lagrangians will be solved in other papers. The first

$$\mathcal{L} = -\partial_\mu \bar{\psi}_1 \gamma_{\mu\nu} \partial_\nu \psi_1 - \partial_\mu \bar{\psi}_2 \gamma_{\mu\nu} \partial_\nu \psi_2 - m^2 \bar{\psi}_1 \psi_1 + m^2 \bar{\psi}_2 \psi_2 \quad . \quad (75)$$

The Lagrangian (75) leads to the equations (64a,64b) which possess solutions with a “correct” (bradyon) physical dispersion and tachyonic solutions as well. This Lagrangian (75) is scalar, Hermitian and it contains only first-order time derivatives. In order to obtain Lagrangians corresponding to the Tucker-Hammer set (57,59), obviously, one should add in (75) terms answering for the Klein-Gordon equation.

At this point I would like to regard the question of solutions in the momentum space. Using the plane-wave expansion

$$\psi_1(x) = \sum_\sigma \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{m\sqrt{2E_p}} \left[\mathcal{U}_1^\sigma(\mathbf{p}) a_\sigma(\mathbf{p}) e^{ip \cdot x} + \mathcal{V}_1^\sigma(\mathbf{p}) b_\sigma^\dagger(\mathbf{p}) e^{-ip \cdot x} \right] \quad , \quad (76a)$$

$$\psi_2(x) = \sum_\sigma \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{m\sqrt{2E_p}} \left[\mathcal{U}_2^\sigma(\mathbf{p}) c_\sigma(\mathbf{p}) e^{ip \cdot x} + \mathcal{V}_2^\sigma(\mathbf{p}) d_\sigma^\dagger(\mathbf{p}) e^{-ip \cdot x} \right] \quad , \quad (76b)$$

one can see that the momentum-space ‘double’ equations

$$\left[-\gamma_{44} E^2 + 2iE\gamma_{4i}\mathbf{p}_i + \gamma_{ij}\mathbf{p}_i\mathbf{p}_j + m^2 \right] \mathcal{U}_1^\sigma(\mathbf{p}) = 0 \quad (\text{or } \mathcal{V}_1^\sigma(\mathbf{p})) \quad , \quad (77a)$$

$$\left[-\gamma_{44} E^2 + 2iE\gamma_{4i}\mathbf{p}_i + \gamma_{ij}\mathbf{p}_i\mathbf{p}_j - m^2 \right] \mathcal{U}_2^\sigma(\mathbf{p}) = 0 \quad (\text{or } \mathcal{V}_2^\sigma(\mathbf{p})) \quad (77b)$$

are satisfied by bispinors

$$\mathcal{U}_1^{(1)\sigma}(\mathbf{p}) = \frac{m}{\sqrt{2}} \left(\begin{bmatrix} 1 + \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(E_p+m)} \\ 1 - \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(E_p+m)} \end{bmatrix} \xi_\sigma \right) \quad , \quad (78)$$

and

$$\mathcal{U}_2^{(1)\sigma}(\mathbf{p}) = \frac{m}{\sqrt{2}} \left(\begin{bmatrix} 1 + \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(E_p+m)} \\ -1 + \frac{(\mathbf{J} \cdot \mathbf{p})}{m} - \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(E_p+m)} \end{bmatrix} \xi_\sigma \right) \quad , \quad (79)$$

respectively. The form (78) has been presented by Hammer, Tucker and Novozhilov in refs. [69,54]. The bispinor normalization in most of the previous papers is chosen to unit. However, as mentioned in ref. [3] it is more convenient to work with bispinors normalized to the mass, *e.g.*, $\pm m^{2j}$ in order to make zero-momentum spinors to vanish in the massless

alternative form is with the following dynamical part:

$$\mathcal{L}^{(2')} = -\partial_\mu \bar{\psi}_1 \gamma_{\mu\nu} \partial_\nu \psi_2 - \partial_\mu \bar{\psi}_2 \gamma_{\mu\nu} \partial_\nu \psi_1 \quad , \quad (73)$$

where ψ_1 and ψ_2 are defined by the equations (64a,64b). This form appear not to admit the mass term in an ordinary sense. The second Lagrangian composed of the two states (ψ_1 , the solution of the equation (64a) and ψ_2 , of the equation (66b), *e.g.*,

$$\mathcal{L}^{(2'')} = -\partial_\mu \bar{\psi}_1 \tilde{\gamma}_{\mu\nu} \partial_\nu \psi_2 - \partial_\mu \bar{\psi}_2 \gamma_{\mu\nu} \partial_\nu \psi_1 - m^2 \bar{\psi}_2 \psi_1 - m^2 \bar{\psi}_1 \psi_2 \quad (74)$$

is not Hermitian.

limit. Here and below I keep the normalization of bispinors as in ref. [3]. Bispinors of Ahluwalia *et al.*, ref. [3], can be written in the more compact form:

$$u_{AJG}^\sigma(\mathbf{p}) = \begin{pmatrix} \left[m + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{E_p + m} \right] \xi_\sigma \\ (\mathbf{J} \cdot \mathbf{p}) \xi_\sigma \end{pmatrix} \quad , \quad v_{AJG}^\sigma(\mathbf{p}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} u_{AJG}^\sigma(\mathbf{p}) \quad . \quad (80)$$

They coincide with the Hammer-Tucker-Novozhilov bispinors within a normalization and a unitary transformation by \mathbf{U} matrix:

$$u_{[3]}^\sigma(\mathbf{p}) = m \cdot \mathbf{U} \mathcal{U}_{[69,54]}^\sigma(\mathbf{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathcal{U}_{[69,54]}^\sigma(\mathbf{p}) \quad , \quad (81a)$$

$$v_{[3]}^\sigma(\mathbf{p}) = m \cdot \mathbf{U} \gamma_5 \mathcal{U}_{[69,54]}^\sigma(\mathbf{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \gamma_5 \mathcal{U}_{[69,54]}^\sigma(\mathbf{p}) \quad . \quad (81b)$$

But, as we found the Weinberg equations (with $+m^2$ and with $-m^2$) have solutions with both positive- and negative-energies. We have to propose the interpretation of the latter. In the framework of this paper one can consider that

$$\mathcal{V}_\sigma^{(1)}(\mathbf{p}) = (-1)^{1-\sigma} \gamma_5 S_{[1]}^c \mathcal{U}_{-\sigma}^{(1)}(\mathbf{p}) \quad (82)$$

and, thus, the explicit forms of the negative-energy solutions would be the same as those of the positive-energy solutions in accordance with definitions (47,48). Thus, in the case of the choice $\mathcal{U}_1^{(1)\sigma}(\mathbf{p})$ and $\mathcal{V}_2^{(1)\sigma}(\mathbf{p}) \sim \gamma_5 \mathcal{U}_1^{(1)\sigma}(\mathbf{p})$ as physical bispinors we come to the Bargmann-Wightman-Wigner-type (BWW) quantum field model proposed by Ahluwalia *et al.* Of course, following the same logic one can choose $\mathcal{U}_2^{(1)\sigma}$ and $\mathcal{V}_1^{(1)\sigma}$ as positive- and negative- bispinors, respectively, and come to a reformulation of the BWW theory. While in this case parities of a boson and its antiboson are opposite, we have -1 for \mathcal{U} - bispinor and $+1$ for \mathcal{V} - bispinor, *i.e.* different in the sign from the model of Ahluwalia *et al.*¹⁰ In the meantime, the construct proposed by Weinberg [71] and developed in this paper is also possible. The $\mathcal{V}_1^{(1)\sigma}(\vec{p})$ as defined by (82) can also be solutions of the equation (64a). The origin of the possibility that the positive- and negative-energy solutions in Eqs. (77a,77b) can coincide each other is the following: the Weinberg equations are of the second order in time derivatives. The Bargmann-Wightman-Wigner construct presented by Ahluwalia [3] is not the only construct in the $(1, 0) \oplus (0, 1)$ representation and one can start with the earlier definitions of the $2(2j + 1)$ bispinors.

Next, previously we gave two additional equations (66a,66b). Their solutions can also be useful because of the possibility of the use of different Lagrangian forms. Solutions in the momentum representation are written

$$\mathcal{U}_1^{(2)\sigma}(\mathbf{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} \left[1 - \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(E_p + m)} \right] \xi_\sigma \\ \left[-1 - \frac{(\mathbf{J} \cdot \mathbf{p})}{m} - \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(E_p + m)} \right] \xi_\sigma \end{pmatrix} \quad , \quad (83)$$

$$\mathcal{U}_2^{(2)\sigma}(\mathbf{p}) = \frac{m}{\sqrt{2}} \begin{pmatrix} \left[1 - \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(E_p + m)} \right] \xi_\sigma \\ \left[1 + \frac{(\mathbf{J} \cdot \mathbf{p})}{m} + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{m(E_p + m)} \right] \xi_\sigma \end{pmatrix} \quad . \quad (84)$$

¹⁰At the present level of our knowledge this mathematical difference has no physical significance, but we want to stay at the most general positions. Perhaps, some yet unknown forms of interactions (*e.g.* of neutral particles) can lead to the observed physical difference between these models.

Therefore, one has $\mathcal{U}_2^{(1)}(\mathbf{p}) = \gamma_5 \mathcal{U}_1^{(1)}(\mathbf{p})$ and $\bar{\mathcal{U}}_2^{(1)}(\mathbf{p}) = -\bar{\mathcal{U}}_1^{(1)}(\mathbf{p})\gamma_5$; $\mathcal{U}_1^{(2)}(\mathbf{p}) = \gamma_5 \gamma_{44} \mathcal{U}_1^{(1)}(\mathbf{p})$ and $\bar{\mathcal{U}}_1^{(2)} = \bar{\mathcal{U}}_1^{(1)} \gamma_5 \gamma_{44}$; $\mathcal{U}_2^{(2)}(\mathbf{p}) = \gamma_{44} \mathcal{U}_1^{(1)}(\mathbf{p})$ and $\bar{\mathcal{U}}_2^{(2)}(\mathbf{p}) = \bar{\mathcal{U}}_1^{(1)} \gamma_{44}$. In fact, they are connected by the transformations of the inversion group.

Let me now apply the quantization procedure to the Weinberg fields. From the definitions [47]:

$$\mathcal{T}_{\mu\nu} = - \sum_i \left\{ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \partial_\nu \phi_i + \partial_\nu \bar{\phi}_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\phi}_i)} \right\} + \mathcal{L} \delta_{\mu\nu} \quad , \quad (85a)$$

$$P_\mu = \int \mathcal{P}_\mu(x) d^3x = -i \int \mathcal{T}_{4\mu} d^3x \quad (85b)$$

one can find the energy-momentum tensor

$$\begin{aligned} \mathcal{T}_{\mu\nu} = & \partial_\alpha \bar{\psi}_1 \gamma_{\alpha\mu} \partial_\nu \psi_1 + \partial_\nu \bar{\psi}_1 \gamma_{\mu\alpha} \partial_\alpha \psi_1 + \\ & + \partial_\alpha \bar{\psi}_2 \gamma_{\alpha\mu} \partial_\nu \psi_2 + \partial_\nu \bar{\psi}_2 \gamma_{\mu\alpha} \partial_\alpha \psi_2 + \mathcal{L} \delta_{\mu\nu} \quad . \end{aligned} \quad (86)$$

As a result the Hamiltonian is

$$\begin{aligned} \mathcal{H} = & \int \left[-\partial_4 \bar{\psi}_2 \gamma_{44} \partial_4 \psi_2 + \partial_i \bar{\psi}_2 \gamma_{ij} \partial_j \psi_2 - \right. \\ & \left. - \partial_4 \bar{\psi}_1 \gamma_{44} \partial_4 \psi_1 + \partial_i \bar{\psi}_1 \gamma_{ij} \partial_j \psi_1 + m^2 \bar{\psi}_1 \psi_1 - m^2 \bar{\psi}_2 \psi_2 \right] d^3x \quad . \end{aligned} \quad (87)$$

Using the plane-wave expansion and the procedure of, *e.g.*, ref. [13] one can come to the quantized Hamiltonian

$$\mathcal{H} = \sum_\sigma \int \frac{d^3\mathbf{p}}{(2\pi)^3} E_p \left[a_\sigma^\dagger(\mathbf{p}) a_\sigma(\mathbf{p}) + b_\sigma(\mathbf{p}) b_\sigma^\dagger(\mathbf{p}) + c_\sigma^\dagger(\mathbf{p}) c_\sigma(\mathbf{p}) + d_\sigma(\mathbf{p}) d_\sigma^\dagger(\mathbf{p}) \right] \quad , \quad (88)$$

Therefore, following the standard textbooks, *e.g.*, refs. [13,12] the commutation relations can be set up as follows:

$$\left[a_\sigma(\mathbf{p}), a_{\sigma'}^\dagger(\mathbf{k}) \right]_- = \left[c_\sigma(\mathbf{p}), c_{\sigma'}^\dagger(\mathbf{k}) \right]_- = (2\pi)^3 \delta_{\sigma\sigma'} \delta(\mathbf{p} - \mathbf{k}) \quad , \quad (89a)$$

$$\left[b_\sigma(\mathbf{p}), b_{\sigma'}^\dagger(\mathbf{k}) \right]_- = \left[d_\sigma(\mathbf{p}), d_{\sigma'}^\dagger(\mathbf{k}) \right]_- = (2\pi)^3 \delta_{\sigma\sigma'} \delta(\mathbf{p} - \mathbf{k}) \quad , \quad (89b)$$

or even in the more general form [25,29]. It is easy to see that the Hamiltonian is positive-definite and the translational invariance still keeps in the framework of this description (*cf.* with refs. [71,3]). Please pay attention here: *I did never apply the indefinite metric.*

Analogously, from the definitions

$$\mathcal{J}_\mu = -i \sum_i \left\{ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \phi_i - \bar{\phi}_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \bar{\phi}_i)} \right\} \quad , \quad (90a)$$

$$Q = -i \int \mathcal{J}_4(x) d^3x \quad , \quad (90b)$$

and

$$\begin{aligned} \mathcal{M}_{\mu\nu,\lambda} = & x_\mu \mathcal{T}_{\lambda\nu} - x_\nu \mathcal{T}_{\lambda\mu} - \\ & - i \sum_i \left\{ \frac{\partial \mathcal{L}}{\partial(\partial_\lambda \phi_i)} N_{\mu\nu}^{\phi_i} \phi_i + \bar{\phi}_i N_{\mu\nu}^{\bar{\phi}_i} \frac{\partial \mathcal{L}}{\partial(\partial_\lambda \bar{\phi}_i)} \right\} \quad , \end{aligned} \quad (91a)$$

$$M_{\mu\nu} = -i \int \mathcal{M}_{\mu\nu,4}(x) d^3x \quad , \quad (91b)$$

one can find the current operator

$$\begin{aligned} \mathcal{J}_\mu = i \Big[& \partial_\alpha \bar{\psi}_1 \gamma_{\alpha\mu} \psi_1 - \bar{\psi}_1 \gamma_{\mu\alpha} \partial_\alpha \psi_1 + \\ & + \partial_\alpha \bar{\psi}_2 \gamma_{\alpha\mu} \psi_2 - \bar{\psi}_2 \gamma_{\mu\alpha} \partial_\alpha \psi_2 \Big] \quad , \end{aligned} \quad (92)$$

and using (91a,91b) the spin momentum tensor

$$\begin{aligned} S_{\mu\nu,\lambda} = i \Big[& \partial_\alpha \bar{\psi}_1 \gamma_{\alpha\lambda} N_{\mu\nu}^{\psi_1} \psi_1 + \bar{\psi}_1 N_{\mu\nu}^{\bar{\psi}_1} \gamma_{\lambda\alpha} \partial_\alpha \psi_1 + \\ & + \partial_\alpha \bar{\psi}_2 \gamma_{\alpha\lambda} N_{\mu\nu}^{\psi_2} \psi_2 + \bar{\psi}_2 N_{\mu\nu}^{\bar{\psi}_2} \gamma_{\lambda\alpha} \partial_\alpha \psi_2 \Big] \quad . \end{aligned} \quad (93)$$

If the Lorentz group generators (a $j = 1$ case) are defined from

$$\bar{\Lambda} \gamma_{\mu\nu} \Lambda a_{\mu\alpha} a_{\nu\beta} = \gamma_{\alpha\beta} \quad , \quad (94a)$$

$$\bar{\Lambda} \Lambda = 1 \quad , \quad (94b)$$

$$\bar{\Lambda} = \gamma_{44} \Lambda^\dagger \gamma_{44} \quad . \quad (94c)$$

then in order to keep the Lorentz covariance of the Weinberg equations and of the Lagrangian (75) one can use the following generators:

$$N_{\mu\nu}^{\psi_1, \psi_2(j=1)} = -N_{\mu\nu}^{\bar{\psi}_1, \bar{\psi}_2(j=1)} = \frac{1}{6} \gamma_{5,\mu\nu} \quad , \quad (95)$$

The matrix $\gamma_{5,\mu\nu} = i [\gamma_{\mu\lambda}, \gamma_{\nu\lambda}]_-$ is defined to be Hermitian.

The quantized charge operator and the quantized spin operator follow immediately from (92) and (93):

$$Q = \sum_\sigma \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \left[a_\sigma^\dagger(\mathbf{p}) a_\sigma(\mathbf{p}) - b_\sigma(\mathbf{p}) b_\sigma^\dagger(\mathbf{p}) + c_\sigma^\dagger(\mathbf{p}) c_\sigma(\mathbf{p}) - d_\sigma(\mathbf{p}) d_\sigma^\dagger(\mathbf{p}) \right] \quad , \quad (96)$$

$$\begin{aligned} (W \cdot n)/m = \sum_{\sigma\sigma'} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{1}{m^2 E_p} \bar{\mathcal{U}}_1^\sigma(\mathbf{p}) (E_p \gamma_{44} - i \gamma_{4i} p_i) I \otimes (\mathbf{J} \cdot \mathbf{n}) \mathcal{U}_1^{\sigma'}(\mathbf{p}) \times \\ \times \left[a_\sigma^\dagger(\mathbf{p}) a_{\sigma'}(\mathbf{p}) + c_\sigma^\dagger(\mathbf{p}) c_{\sigma'}(\mathbf{p}) - b_\sigma(\mathbf{p}) b_{\sigma'}^\dagger(\mathbf{p}) - d_\sigma(\mathbf{p}) d_{\sigma'}^\dagger(\mathbf{p}) \right] \end{aligned} \quad (97)$$

(provided that a frame is chosen in such a way that $\mathbf{n} \parallel \mathbf{p}$ is along the third axis). It is easy to verify eigenvalues of the charge operator are ± 1 ,¹¹ and of the Pauli-Lyuban'sky spin operator are

$$\xi_\sigma^*(\mathbf{J} \cdot \mathbf{n}) \xi_{\sigma'} = +1, 0 - 1 \quad (98)$$

in a massive case and ± 1 in a massless case (see the discussion on the massless limit of the Weinberg bispinors in ref. [3a]).

In order to solve the question of finding propagators in this theory we would like to consider the most general case. In ref. [3a] a particular case of the BWB bispinors has been

¹¹In the Majorana construct the eigenvalue is zero.

regarded. In order to decide what case is physically relevant for describing one or another situation one should know what symmetries are respected by the Nature. So, let me check, if the sum of four equations ($x = x_2 - x_1$)

$$\begin{aligned}
& \left[\gamma_{\mu\nu} \partial_\mu \partial_\nu - m^2 \right] \int \frac{d^3 \mathbf{p}}{(2\pi)^3 2E_p} \left[a \theta(t_2 - t_1) \mathcal{U}_1^{\sigma(1)}(\mathbf{p}) \otimes \overline{\mathcal{U}}_1^{\sigma(1)}(\mathbf{p}) e^{ip \cdot x} + \right. \\
& \quad \left. + b \theta(t_1 - t_2) \mathcal{V}_1^{\sigma(1)}(\mathbf{p}) \otimes \overline{\mathcal{V}}_1^{\sigma(1)}(\mathbf{p}) e^{-ip \cdot x} \right] + \\
& + \left[\gamma_{\mu\nu} \partial_\mu \partial_\nu + m^2 \right] \int \frac{d^3 \mathbf{p}}{(2\pi)^3 2E_p} \left[a \theta(t_2 - t_1) \mathcal{U}_2^{\sigma(1)}(\mathbf{p}) \otimes \overline{\mathcal{U}}_2^{\sigma(1)}(\mathbf{p}) e^{ip \cdot x} + \right. \\
& \quad \left. + b \theta(t_1 - t_2) \mathcal{V}_2^{\sigma(1)}(\mathbf{p}) \otimes \overline{\mathcal{V}}_2^{\sigma(1)}(\mathbf{p}) e^{-ip \cdot x} \right] + \\
& + \left[\tilde{\gamma}_{\mu\nu} \partial_\mu \partial_\nu + m^2 \right] \int \frac{d^3 \mathbf{p}}{(2\pi)^3 2E_p} \left[a \theta(t_2 - t_1) \mathcal{U}_1^{\sigma(2)}(\mathbf{p}) \otimes \overline{\mathcal{U}}_1^{\sigma(2)}(\mathbf{p}) e^{ip \cdot x} + \right. \\
& \quad \left. + b \theta(t_1 - t_2) \mathcal{V}_1^{\sigma(2)}(\mathbf{p}) \otimes \overline{\mathcal{V}}_1^{\sigma(2)}(\mathbf{p}) e^{-ip \cdot x} \right] + \\
& + \left[\tilde{\gamma}_{\mu\nu} \partial_\mu \partial_\nu - m^2 \right] \int \frac{d^3 \mathbf{p}}{(2\pi)^3 2E_p} \left[a \theta(t_2 - t_1) \mathcal{U}_2^{\sigma(2)}(\mathbf{p}) \otimes \overline{\mathcal{U}}_2^{\sigma(2)}(\mathbf{p}) e^{ip \cdot x} + \right. \\
& \quad \left. + b \theta(t_1 - t_2) \mathcal{V}_2^{\sigma(2)}(\mathbf{p}) \otimes \overline{\mathcal{V}}_2^{\sigma(2)}(\mathbf{p}) e^{-ip \cdot x} \right] = \delta^{(4)}(x_2 - x_1) \quad (99)
\end{aligned}$$

can be satisfied by the definite choice of the constant a and b . In the process of calculations I assume that the set of the analogs of the “Pauli spinors” in the $(1, 0)$ or $(0, 1)$ spaces is a complete set in mathematical sense and it is normalized to $\delta_{\sigma\sigma'}$. Simple calculations yield

$$\begin{aligned}
& \partial_\mu \partial_\nu \left[a \theta(t_2 - t_1) e^{ip \cdot (x_2 - x_1)} + b \theta(t_1 - t_2) e^{-ip \cdot (x_2 - x_1)} \right] = \\
& = - \left[a p_\mu p_\nu \theta(t_2 - t_1) \exp[ip \cdot (x_2 - x_1)] + b p_\mu p_\nu \theta(t_1 - t_2) \exp[-ip \cdot (x_2 - x_1)] \right] + \\
& + a \left[-\delta_{\mu 4} \delta_{\nu 4} \delta'(t_2 - t_1) + i(p_\mu \delta_{\nu 4} + p_\nu \delta_{\mu 4}) \delta(t_2 - t_1) \right] \exp[i\mathbf{p}(\mathbf{x}_2 - \mathbf{x}_1)] + \\
& + b \left[\delta_{\mu 4} \delta_{\nu 4} \delta'(t_2 - t_1) + i(p_\mu \delta_{\nu 4} + p_\nu \delta_{\mu 4}) \delta(t_2 - t_1) \right] \exp[-i\mathbf{p}(\mathbf{x}_2 - \mathbf{x}_1)] \quad ; \quad (100)
\end{aligned}$$

and

$$\mathcal{U}_1^{(1)} \overline{\mathcal{U}}_1^{(1)} = \frac{1}{2} \begin{pmatrix} m^2 & S_p \otimes S_p \\ \overline{S}_p \otimes \overline{S}_p & m^2 \end{pmatrix} \quad , \quad \mathcal{U}_2^{(1)} \overline{\mathcal{U}}_2^{(1)} = \frac{1}{2} \begin{pmatrix} -m^2 & S_p \otimes S_p \\ \overline{S}_p \otimes \overline{S}_p & -m^2 \end{pmatrix} \quad , \quad (101)$$

$$\mathcal{U}_1^{(2)} \overline{\mathcal{U}}_1^{(2)} = \frac{1}{2} \begin{pmatrix} -m^2 & \overline{S}_p \otimes \overline{S}_p \\ S_p \otimes S_p & -m^2 \end{pmatrix} \quad , \quad \mathcal{U}_2^{(2)} \overline{\mathcal{U}}_2^{(2)} = \frac{1}{2} \begin{pmatrix} m^2 & \overline{S}_p \otimes \overline{S}_p \\ S_p \otimes S_p & m^2 \end{pmatrix} \quad , \quad (102)$$

where

$$S_p = m + (\mathbf{J} \cdot \mathbf{p}) + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{E_p + m} \quad , \quad (103)$$

$$\overline{S}_p = m - (\mathbf{J} \cdot \mathbf{p}) + \frac{(\mathbf{J} \cdot \mathbf{p})^2}{E_p + m} \quad . \quad (104)$$

Due to

$$[E_p - (\mathbf{J} \cdot \mathbf{p})] S_p \otimes S_p = m^2 [E_p + (\mathbf{J} \cdot \mathbf{p})] \quad ,$$

$$[E_p + (\mathbf{J} \cdot \mathbf{p})] \bar{S}_p \otimes \bar{S}_p = m^2 [E_p - (\mathbf{J} \cdot \mathbf{p})]$$

after simplifying the left side of (99) and comparing it with the right side we find the constants to be equal to $a = b = 1/4im^2$. Thus, if consider all four equations (64a,64b,66a,66b) one can use the “Wick’s formula” for the time-ordered particle operators to find propagators:

$$S_F^{(1)}(p) = \frac{i [\gamma_{\mu\nu} p_\mu p_\nu - m^2]}{(2\pi)^4(p^2 + m^2 - i\epsilon)} \quad , \quad (105a)$$

$$S_F^{(2)}(p) = \frac{i [\gamma_{\mu\nu} p_\mu p_\nu + m^2]}{(2\pi)^4(p^2 + m^2 - i\epsilon)} \quad , \quad (105b)$$

$$S_F^{(3)}(p) = \frac{i [\tilde{\gamma}_{\mu\nu} p_\mu p_\nu + m^2]}{(2\pi)^4(p^2 + m^2 - i\epsilon)} \quad , \quad (105c)$$

$$S_F^{(4)}(p) = \frac{i [\tilde{\gamma}_{\mu\nu} p_\mu p_\nu - m^2]}{(2\pi)^4(p^2 + m^2 - i\epsilon)} \quad . \quad (105d)$$

The conclusion is that the states described by the equations (64a,64b,66a,66b) cannot propagate separately each other, what is a principal difference comparing with the Dirac case.

Furthermore, I am able to recast the $j = 1$ Tucker-Hammer equation (57) which is free of tachyonic solutions, or the Proca equation (52), to the form

$$m^2 E_i = -\frac{\partial^2 E_i}{\partial t^2} + \epsilon_{ijk} \frac{\partial}{\partial x_j} \frac{\partial B_k}{\partial t} + \frac{\partial}{\partial x_i} \frac{\partial E_j}{\partial x_j} \quad , \quad (106)$$

$$m^2 B_i = \epsilon_{ijk} \frac{\partial}{\partial x_j} \frac{\partial E_k}{\partial t} + \frac{\partial^2 B_i}{\partial x_j^2} - \frac{\partial}{\partial x_i} \frac{\partial B_j}{\partial x_j} \quad . \quad (107)$$

The Klein-Gordon equation (the D’Alembert equation in the massless limit)

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i^2} \right) F_{\mu\nu} = -m^2 F_{\mu\nu} \quad (108)$$

is implied ($c = \hbar = 1$). Introducing vector operators one can write equations in the following form:

$$\frac{\partial}{\partial t} \text{curl } \mathbf{B} + \text{grad div } \mathbf{E} - \frac{\partial^2 \mathbf{E}}{\partial t^2} = m^2 \mathbf{E} \quad , \quad (109a)$$

$$\nabla^2 \mathbf{B} - \text{grad div } \mathbf{B} + \frac{\partial}{\partial t} \text{curl } \mathbf{E} = m^2 \mathbf{B} \quad . \quad (109b)$$

Taking into account the definitions:

$$\rho_e = \text{div } \mathbf{E} \quad , \quad \mathbf{J}_e = \text{curl } \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} \quad , \quad (110a)$$

$$\rho_m = \text{div } \mathbf{B} \quad , \quad \mathbf{J}_m = -\frac{\partial \mathbf{B}}{\partial t} - \text{curl } \mathbf{E} \quad , \quad (110b)$$

the relations of the vector algebra (\mathbf{X} is an arbitrary vector):

$$\text{curl curl } \mathbf{X} = \text{grad div } \mathbf{X} - \nabla^2 \mathbf{X} \quad , \quad (111)$$

and the Klein-Gordon equation (108) one obtains two equivalent sets of equations, which complete the Maxwell's set of equations. The first one is

$$\frac{\partial \mathbf{J}_e}{\partial t} + \text{grad } \rho_e = m^2 \mathbf{E} \quad , \quad (112a)$$

$$\frac{\partial \mathbf{J}_m}{\partial t} + \text{grad } \rho_m = 0 \quad ; \quad (112b)$$

and the second one is

$$\text{curl } \mathbf{J}_m = 0 \quad (113a)$$

$$\text{curl } \mathbf{J}_e = -m^2 \mathbf{B} \quad . \quad (113b)$$

I would like to remind that the Weinberg equations (and, hence, the equations (112a-113b))¹² can be obtained on the basis of a very few number of postulates; in fact, by using the Lorentz transformation rules for the Weinberg bivector (or for the antisymmetric tensor field) and the Ryder-Burgard relation [3,5].

In a massless limit the situation is different. Firstly, the set of equations (110b), with the left side are chosen to be zero, is “an identity satisfied by certain space-time derivatives of $F_{\mu\nu} \dots$, namely, refs. [30,68]

$$\frac{\partial F_{\mu\nu}}{\partial x^\sigma} + \frac{\partial F_{\nu\sigma}}{\partial x^\mu} + \frac{\partial F_{\sigma\mu}}{\partial x^\nu} = 0 \quad .” \quad (116)$$

I believe that a similar consideration for the dual field $\tilde{F}_{\mu\nu}$ as in refs. [30,68] can reveal that the same is true for the first equations (110a). So, in the massless case we met with the problem of interpretation of the charge and the current.

Secondly, in order to satisfy the massless equations (113a,113b) one should assume that the currents are represented in gradient forms of some scalar fields $\chi_{e,m}$. What physical significance should be attached to these chi-functions? In a massless case the charge densities are then (see equations (112a,112b))

$$\rho_e = -\frac{\partial \chi_e}{\partial t} + \text{const} \quad , \quad \rho_m = -\frac{\partial \chi_m}{\partial t} + \text{const} \quad , \quad (117)$$

¹²Beginning from the dual massive equations (59,61) and setting $\mathbf{C} \equiv \mathbf{E}$, $\mathbf{D} \equiv \mathbf{B}$ one could obtain

$$\frac{\partial \mathbf{J}_e}{\partial t} + \text{grad } \rho_e = 0 \quad , \quad (114a)$$

$$\frac{\partial \mathbf{J}_m}{\partial t} + \text{grad } \rho_m = m^2 \mathbf{B} \quad ; \quad (114b)$$

and

$$\text{curl } \mathbf{J}_e = 0 \quad , \quad (115a)$$

$$\text{curl } \mathbf{J}_m = m^2 \mathbf{E} \quad . \quad (115b)$$

This signifies that the physical content spanned by massive dual fields can be different. The reader can easily reveal parity-conjugated equations from Eqs. (66a,66b).

what tells us that ρ_e and ρ_m are constants provided that the primary functions $\chi_{e,m}$ are linear functions in time (decreasing or increasing?). One can obtain the Maxwell's free-space equations, in the definite choice of the χ_e and χ_m , namely, in the case when they are constants.

It is useful to compare the resulting equations for $\rho_{e,m}$ and $\mathbf{J}_{e,m}$ and the fact of appearance of functions $\chi_{e,m}$ with alternative formulations of electromagnetic theory discussed in the Section I. I believe, this concept can also be useful in analyses of the $E = 0$ solutions in higher-spin relativistic wave equations [58,49,1,2], which have been “baptized” by Moshinsky and Del Sol in [53] as the ‘relativistic cockroach nest’. Finally, in ref. [35] it was mentioned that solutions of Eqs. (4.21,4.22) of ref. [71b], see the same equations (2a,2b) in this article, satisfy the equations of the type (106,107), “*but not always vice versa*”. An interpretation of this statement and investigations of Eq. (57) with other initial and boundary conditions (or of the functions χ) deserve further elaboration (both theoretical and experimental).

IV. CONCLUSION

As a conclusion of a serie of papers of mine in various journals (here I present only a part of these investigations), one can state that due to the present-day experimental situation the standard model seems to be able to describe a restricted class of phenomena only. Probably, origins of these limitations are in methodological failures which were brought as a result of the unreasonable (and everywhere) application of the principles which the Maxwell's electromagnetic theory is based on. In my opinion, it is a particular case only. Generalized models discussed in this paper appear to be suitable candidates to begin to work out the unified field theory. All they are connected each other and it seems to indicate at the same physical reality. I believe the Weinberg $2(2j + 1)$ component formalism is the most convenient way for understanding the nature of higher spin particles, the structure of the space-time and for describing many processes with particles of the spin 0 and 1, because this formalism is on an equal footing with the well-developed Dirac formalism for spin-1/2 charged particles and manifests explicitly those symmetry properties which are related with the Poincarè group and the group of inversion operations. The problems in Majorana-like constructs in both the $(1/2, 0) \oplus (0, 1/2)$ representation and higher-spin representations still deserve further elaboration.

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